COMPUTER SCIENCE & ENGINEERING

QUESTION BANK

Course Title: DISCRETE MATHEMATICS
Course Code: 25CS301
Regulation: NR25

COURSE OBJECTIVES

- Introduces elementary discrete mathematics for Computer Science and Engineering.
- Topics include formal logic notation, methods of proof, induction, sets, relations, graph theory, permutations and combinations, counting principles, recurrence relations and generating functions.

COURSE OUTCOMES

- **CO1:** Ability to understand and construct precise mathematical proofs.
- **CO2:** Ability to use logic and set theory to formulate precise statements.
- **CO3:** Ability to analyze and solve counting problems on finite and discrete structures.
- **CO4:** Ability to describe and manipulate sequences.
- **CO5:** Ability to apply graph theory in solving computing problems.

UNIT – III

ALGEBRAIC STRUCTURES

Syllabus: Introduction, Algebraic Systems, Semigroups and Monoids, Lattices as Partially Ordered Sets, Boolean Algebra.

PART – A

SHORT ANSWER QUESTIONS

S.No.	Questions	BT	CO	PO
1	Define an algebraic structure. Give a suitable example.	L1	CO3	PO1
2	Define a binary operation on a non-empty set and give an example.	L1	CO3	PO1
3	Define a semigroup. Give a suitable example.	L1	CO3	PO1
4	Define a monoid. Give a suitable example.	L1	CO3	PO1
5	Define a commutative semigroup and give an example.	L1	CO3	PO1
6	State the necessary conditions for an algebraic structure $(S, *)$ to be a semigroup.	L1	CO3	PO1
7	State the necessary conditions for an algebraic structure $(S, *)$ to be a monoid.	L1	CO3	PO1
8	Define an idempotent element with respect to a binary operation and give an example.	L1	CO3	PO1
9	Define a lattice as a partially ordered set.	L1	CO3	PO1
10	Define the meet and join of two elements in a lattice.	L1	CO3	PO1
11	Define a bounded lattice and give an example.	L1	CO3	PO1
12	Define a complemented lattice and give an example.	L1	CO3	PO1
13	Define a distributive lattice.	L1	CO3	PO1
14	Define a modular lattice.	L1	CO3	PO1
15	State the idempotent laws of Boolean algebra.	L1	CO3	PO1
16	State the complement laws of Boolean algebra.	L1	CO3	PO1
17	State the absorption laws of Boolean algebra.	L1	CO3	PO1
18	State De Morgan's laws of Boolean algebra.	L1	CO3	PO1

PART – B

LONG ANSWER QUESTIONS

S.No.	Questions	BT	CO	PO
19	Show that the operation $*$ defined on $\mathbb{R}$ by $a * b = a + b - ab$ is associative. Hence determine whether $(\mathbb{R}, *)$ is a semigroup.	L4	CO3	PO2
20	Show that $(\mathbb{Q}, *)$, where $a * b = a - b + ab$, is not a semigroup by providing a suitable counterexample or by examining the required properties.	L4	CO3	PO2
21	Show that $(\mathbb{N}, -)$ is not a semigroup.	L4	CO3	PO2
22	Let $S = \mathbb{Z}$ and define $a * b = a + b + 1$. Prove that $(S, *)$ is an abelian group. Identify the identity element and the inverse of an arbitrary element a.	L4	CO3	PO2
23	Determine whether the algebraic structure $(\mathbb{Z}, *)$, where $a * b = a + b$, is a semigroup and a monoid. Justify your answer.	L4	CO3	PO2
24	Let $S = \{0, 1, 2, 3\}$ and define $a * b = (a + b) \pmod 4$. Determine whether $(S, *)$ is a semigroup and a monoid.	L4	CO3	PO2
25	Let $S = \{1, -1, i, -i\}$. Show that S forms an algebraic structure under multiplication and verify the required properties.	L4	CO3	PO2
26	Prove that in a monoid, the identity element is unique.	L3	CO3	PO1
27	Prove the cancellation laws in a group: if $a * b = a * c$, then $b = c$, and if $b * a = c * a$, then $b = c$.	L4	CO3	PO2
28	Prove that in a group $(G, *)$, the inverse of every element is unique.	L4	CO3	PO2
29	Let $(L, \leq)$ be a partially ordered set. Define the meet and join of two elements and explain how they lead to the concept of a lattice.	L2	CO3	PO1
30	Draw the Hasse diagram of the divisibility lattice of the positive divisors of 12. Identify the meet and join of suitable pairs of elements.	L3	CO3	PO2
31	Let D_{30} be the set of all positive divisors of 30 ordered by divisibility. Show that $(D_{30}, \mid)$ is a lattice.	L4	CO3	PO2
32	Let D_{42} be the set of all positive divisors of 42 ordered by divisibility. Show that $(D_{42}, \mid)$ is a complemented lattice.	L4	CO3	PO2
33	Find the complements of all elements of the lattice of positive divisors of 30 under the divisibility relation.	L3	CO3	PO2

S.No.	Questions	BT	CO	PO
34	Determine whether the lattice of positive divisors of 12 under divisibility is distributive. Justify your answer.	L4	CO3	PO2
35	Examine whether the lattice represented by a given Hasse diagram is distributive. Use the distributive laws to justify your conclusion.	L4	CO3	PO2
36	Prove that every distributive lattice is modular.	L4	CO3	PO2
37	Prove that a complemented distributive lattice is a Boolean algebra.	L4	CO3	PO2
38	Verify the following Boolean algebra identities: (i) $a + a = a$, (ii) $a \cdot a = a$, (iii) $a + ab = a$, (iv) $a(a + b) = a$.	L3	CO3	PO2
39	Prove De Morgan's laws in Boolean algebra: (i) $(a + b)' = a'b'$, (ii) $(ab)' = a' + b'$.	L4	CO3	PO2
40	Simplify the following Boolean expressions using Boolean algebra laws: (i) $A + AB$, (ii) $A(A + B)$, (iii) $AB + A'B$, (iv) $A + A'B$.	L3	CO3	PO2
41	Verify the principle of duality in Boolean algebra by writing the dual of suitable Boolean expressions and identities.	L3	CO3	PO1
42	Construct the Boolean algebra of subsets of a given universal set and verify the Boolean operations using set operations.	L4	CO3	PO2

BLOOM'S TAXONOMY LEVELS (BT)

Level	Bloom's Taxonomy	Description
L1	Remembering	Define, State, List, Identify, Recall
L2	Understanding	Explain, Describe, Interpret, Classify, Summarize
L3	Applying	Solve, Calculate, Construct, Demonstrate, Use
L4	Analyzing	Analyze, Differentiate, Examine, Compare, Verify, Prove
L5	Evaluating	Justify, Critique, Assess, Validate, Defend
L6	Creating	Design, Formulate, Develop, Generate, Construct

CO–PO ALIGNMENT FOR UNIT-III

CO3

Ability to analyze and solve counting problems on finite and discrete structures.

Note: The stated CO3 specifically refers to counting problems, whereas Unit-III of the supplied NR25 syllabus deals with **Algebraic Structures**. Therefore, the CO mapping provided here follows the existing course-bank convention of mapping Unit-III to **CO3**. For final NBA/OBE documentation, the department may wish to verify whether the official NR25 curriculum has a more appropriate CO mapping for Algebraic Structures.

Unit-III questions assess the ability to:

- Understand algebraic systems and binary operations.
- Identify and verify semigroups and monoids.
- Analyze algebraic properties such as closure and associativity.
- Understand lattices as partially ordered sets.
- Determine meet and join operations.
- Construct and analyze lattice structures using Hasse diagrams.
- Identify bounded and complemented lattices.
- Analyze distributive and modular lattices.
- Apply Boolean algebra laws.
- Simplify Boolean expressions.
- Prove Boolean identities and De Morgan's laws.

PO MAPPING

PO1 – Engineering Knowledge:
Apply mathematical and algebraic principles to solve problems related to Computer Science and Engineering.

PO2 – Problem Analysis:
Analyze algebraic structures, lattices and Boolean expressions and apply appropriate mathematical methods to establish required properties.

UNIT-III TOPIC-WISE COVERAGE

Unit-III Topic	Question Numbers
Algebraic Systems	1, 2, 19, 20, 23, 24, 25
Binary Operations	2, 6, 19, 20, 23, 24
Semigroups	3, 5, 6, 19, 20, 21, 23, 24
Monoids	4, 7, 22, 23, 24, 26
Associativity	6, 19, 20, 21, 23, 24
Lattices as Posets	9, 10, 29, 30, 31

Unit-III Topic	Question Numbers
Meet and Join	10, 29, 30, 31
Bounded Lattices	11, 31
Complemented Lattices	12, 32, 33
Distributive Lattices	13, 34, 35, 36, 37
Modular Lattices	14, 36
Boolean Algebra	15, 16, 17, 18, 37, 38, 39, 40, 41, 42
Boolean Algebra Laws	15–18, 38–41
Boolean Expression Simplification	38, 40
De Morgan's Laws	18, 39

IMPORTANT CORRECTIONS MADE FROM THE ORIGINAL QUESTION BANK

1. **"Show that the operation $a * b = a + b - ab$ is a semigroup"** was changed from **L1 to L4**, because proving associativity requires analysis.

2. **"Show that $(\mathbb{Q}, *)$ is not a semigroup"** was changed from **L1 to L4**, because the student must analyze the algebraic property.

3. **"Define totally ordered set"** was retained as **L1**, but the term belongs to the ordering/lattice discussion.

4. **"What are the idempotent laws of Boolean algebra?"** was changed to **L1**, because it requires recall of the laws.

5. **"State modular lattice"** was changed to **L1**.

6. Questions such as **"Show that $(\mathbb{N}, -)$ is not a semigroup"** are **L4**, not L1.

7. Questions involving proving group/semigroup/monoid properties are classified mainly as **L3/L4**, depending on the mathematical reasoning involved.

8. Lattice questions involving **Hasse diagrams, meet, join, complements and distributivity** have been retained because they are explicitly covered under **"Lattices as Partially Ordered Sets."**

9. Boolean algebra questions have been retained because **Boolean Algebra** is explicitly included in Unit-III.

10. The original Unit-III questions involving groups have been reduced because **Group Theory is not explicitly listed in the Unit-III syllabus statement you provided**. A few basic group questions have been retained only where they naturally support algebraic-structure concepts.

11. **"Complemented lattice" and "distributive lattice"** are correctly treated as Unit-III topics, unlike in the original Unit-II question bank.

12. Questions asking students to **prove, verify, examine or determine properties** have been assigned L3/L4 rather than L1.

RECOMMENDED BT DISTRIBUTION FOR UNIT-III

BT Level	Focus
L1 – Remembering	Definitions, laws and basic terminology
L2 – Understanding	Explanation and interpretation of algebraic concepts
L3 – Applying	Applying algebraic laws, constructing Hasse diagrams and simplifying Boolean expressions
L4 – Analyzing	Proving algebraic properties and analyzing lattices and Boolean structures

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TEXT BOOKS

1. **Discrete Mathematical Structures with Applications to Computer Science** – J. P. Tremblay and R. Manohar, McGraw-Hill, 1st Edition.
2. **Discrete Mathematics for Computer Scientists and Mathematicians** – Joe L. Mott, Abraham Kandel and Theodore P. Baker, Prentice Hall of India, 2nd Edition.

REFERENCE BOOKS

1. **Discrete and Combinatorial Mathematics – An Applied Introduction** – Ralph P. Grimaldi, Pearson Education, 5th Edition.
2. **Discrete Mathematical Structures** – Thomas Koshy, Tata McGraw-Hill.